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No. of pages - 24

111447 (E)

MID-TERM EXAMINATION (2024-25)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**
समय : 3 घंटे **अधिकतम अंक - 80**

सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

1. इस प्रश्न पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
2. यह प्रश्न पत्र पाँच खंडों में विभाजित है - क, ख, ग, घ एवं ङ।
3. खंड-क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न 19 एवं 20 अभिकथन एवं तर्क आधारित एक-एक अंक के प्रश्न हैं।
4. खंड-ख में प्रश्न संख्या 21 से 25 तक अति लघुउत्तरीय (VSA) प्रकार के दो-दो अंक के प्रश्न हैं
5. खंड-ग में प्रश्न संख्या 26 से 31 तक लघुउत्तरीय (SA) प्रकार के तीन-तीन अंक के प्रश्न हैं।
6. खंड-घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच- पाँच अंकों के प्रश्न हैं
7. खंड-ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित चार-चार अंकों के प्रश्न हैं।
8. प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड-ख के 2 प्रश्नों में, खण्ड-ग के प्रश्नों में, खण्ड-घ के 2 प्रश्नों में, खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
9. कैल्कुलेटर का उपयोग वर्जित है।

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section-A, questions No. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section-B, questions No. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section-C, questions No. 26 to 31 are Short answer (SA) type questions, carrying 3 marks each.
6. In Section-D, questions No. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
7. In Section-E, questions No. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section-B, 3 questions in Section-C, 2 questions in Section-D and 2 questions in Section E.
9. Use of calculator is not allowed.

SECTION-A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Let us define a relation R in R as a Rb if $a \geq b$, then R is:

(a) an equivalence relation

(b) reflexive transitive but not symmetric

(c) symmetric transitive but not reflexive

(d) neither transitive nor reflexive but symmetric

2. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matric such that adj A = A, then (a+b+c+d) is equal to : 1

(a) 2a

(b) 2b

(c) 2c

(d) 0

3. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is false?

(a) $a_{11} < 0$

(b) $a_{12} + a_{21} = -6$

(c) $a_{13} > a_{31}$

(d) $a_{31} = 0$

4. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :

(a) 7

(b) 6

(c) 8

(d) 18

5. A function $f : R \to R$ defined as $f(x) = x^2 - 4x + 5$ is : 1

(a) injective but not surjective.

(b) surjective but not injective.

(c) both injective and surjective.

(d) neither injective nor surjective.

6. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then value of $|A(\text{adj.}A)|$ is :

(a) 1001

(b) 101

(c) 10

(d) 1000

7. If A and B are two skew symmetric matrices, then (AB+BA) is :

(a) skew symmetric matrix

(b) a symmetric matrix

(c) a null matrix

(d) an identity matrix

The value of $\left(2\sec^{-1}2 + \sin^{-1}\frac{1}{2}\right)$ is :

(a) $\frac{\pi}{6}$

(b) $\frac{5\pi}{6}$

(c) $\frac{7\pi}{6}$

(d) 1

9. If A and B are two matrices of the order $3 \times m$ and $3 \times n$ and $m = n$, then the order of matrix (5A–2B). 1

(a) $m \times 3$ (b) 3×3

(c) $m \times n$ (d) $3 \times n$

10. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is : 1

(a) [1, 2] (b) [–1, 1]

(c) [0, 1] (d) None of these

11. Function $f(x) = |x| + |x-2|$ is : 1

(a) Continuous but not differentiable at $x = 0$ and $x = 2$

(b) Differentiable but not continuous at $x = 0$ and $x = 2$

(c) Continuous but not differentiable at $x = 0$ only

(d) Neither continuous nor differentiable at $x = 0$ and $x = 2$

12. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is : 1

(a) –60 units/sec (b) 60 units/sec

(c) 70 units/sec (d) –140 units/sec

13. Derivative of $e^{\sin^2 x}$ with respect to cos x is : 1

(a) $\sin x.e^{\sin^2 x}$ (b) $\cos x.e^{\sin^2 x}$

(c) $-2\cos x.e^{\sin^2 x}$ (d) $-2\sin^2 x \cos x.e^{\sin^2 x}$

14. If $\int_0^a \frac{1}{4+x^2}dx = \frac{\pi}{6}$, then the value of 'a' is : 1

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{3}$

(c) $\sqrt{3}$ (d) None of these

15. The degree of differential equation $(y'')^2 + (y')^3 - x\sin(y')$ is : 1

(a) 1 (b) 2

(c) 3 (d) Not defined

16. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is : 1

(a) $e^x + e^{-y} = c$ (b) $e^{-x} + e^{-y} = c$

(c) $e^{x+y} = c$ (d) $2e^{x+y} = c$

17. The integrating factor of the differential equation $(x+2y^2)\frac{dy}{dx} = y(y>0)$ is : 1

(a) $\frac{1}{x}$ (b) x

(c) y (d) $\frac{1}{y}$

18. $\int_{-2}^{3} x^2.dx = k\int_0^2 x^2dx + \int_2^3 x^2dx$, then value of k is : 1

(a) 2 (b) 1

(c) 0 (d) $\frac{1}{2}$

Assertion-Reason based questions

Question number 19 and 20 each carry one mark.

In the following questions a statement of Assertion (A) is followed by statement of Reason (R) is given. Choose the correct answer out of following choices :

(a) **Both A and R are true and R is the correct explanation of A.**

(b) **Both A and Reason R are true but Reason R is not correct explanation of A.**

(c) **A is true but R is false.**

(d) **A is false but R is true.**

19. Assertion (A) : For any symmetric matrix A, B'AB is a skew-symmetric matrix.

Reason (R) : A square matrix P is skew-symmetric if $P' = -P$. 1

20. Assertion (A) : The range of the function $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$. 1

SECTION-B

This section comprises of very short answer (VSA) type questions of 2 marks each.

21. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$ 2

OR

(b) Find the domain of the function $f(x) = \cos^{-1}(1 - x^2)$. Also find its range. 2

22. (a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$. 2

OR

(b) $y = \operatorname{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1+x^2}\frac{dy}{dx} - x = 0$ 2

23. If $P' = \begin{bmatrix} -3 & 5 \\ 3 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$, find $(P + 3Q)'$. 2

24. Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing. 2

25. Evaluate : 2

$$\int_0^{a^3} \frac{x^2}{x^6 + a^6} dx$$

SECTION-C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Show that the relation R on R defined as $R = \{(a,b) : a \leq b\}$ is reflexive and transitive but not symmetric. 3

OR

(b) Let $A = R - \{2\}$ and $B = R - \{1\}$

Consider the function $f : A \to B$ defined by $f(x) = \frac{x-3}{x-2}$. Check whether f is one-one and onto.

27. (a) Find the particular solution of the differential equation given by

$2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$. 3

OR

(b) Solve the following differential equation $x^2dy + y(x + y)dx = 0$ 3

28. (a) If $x^{30}y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$ 3

OR

(b) If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)\frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2$ 3

29. Find : 3

$$\int \frac{2 + \sin 2x}{1 + \cos 2x} e^x dx$$

30. Evaluate : 3

$$\int_{-2}^{2} \sqrt{\frac{2 - x}{2 + x}}\, dx$$

31. On her birthday Myiesha decided to donate some money to the children of an orphanage home If there were 8 children less everyone would have got ₹ 10 more However if there were 16 children more, everyone would have got ₹ 10 less. Using matrix method find the number of children ands amount distributed by Myiesha. 3

SECTION-D

Question number 32 to 35 are long type answer (LA) type. Each carries 5 marks.

32. (a) Show that a function $f : R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to R$ becomes an onto function 5

OR

(b) A relation R is defined on $N \times N$ (where N is the set of natural number) as:

$(a,b)R(c,d) \Leftrightarrow a - c = b - d$

Show that R is an equivalence relation. 5

33. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations: 5

$x + 2y - 3z = 1$
$2x - 3z = 2$
$x + 2y = 3$

OR

(b) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + KI_3 = 0$ then find the value of K and hence find A^{-1}. 5

4. Find : 5

$$\int \frac{(3\cos x - 2)\sin x}{5 - \sin^2 x - 4\cos x}$$

5. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum. 5

SECTION-E

Source based/Case based/passage based/integrated units of assessment questions

6. In a linear algebra, the transpose of a matrix is an operator which flips matrix over its diogonal, i.e. it switches the row and column indices of the matrix A by producing another matrix, often denoted by A^T. The transpose of a matrix was introduced in 1885 by the British mathematician

Transpose

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

If $A = [a_{ij}]_{m \times n}$ be a m × n matrix, then the matrix obtained by interchanging the rows and columns of A is called transpose of A. A square matrix $A = [a_{ij}]$ is said to be symmetric. If $A^T = A$ for all possible values of i and j. A square matrix $A = [a_{ij}]$ is said to be skew symmetric, if $A^T = -A$ for all possible values of i and j.

Based on the above information, answer the following questions:

(i) Find the transpose of $[1 \; -2 \; -5]$

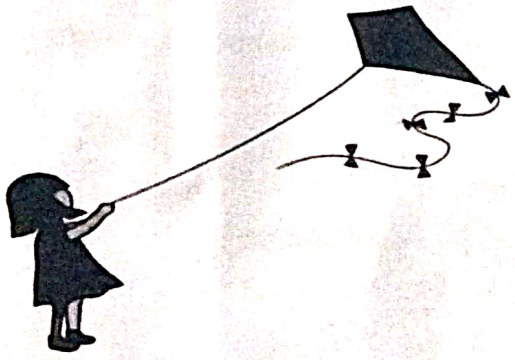
(ii) Find the transpose of matrix (AB)

(iii) Evaluate $(A+B)^T - A$ where $A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

OR

(iii) Evaluate $(AB)^T$ where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

37. Khushali who is student of a prestigious school has a fond of flying kite After completing her Board Exam and CUET Exam she is flying kite to to release the exam stress Her kite is flying along the curve having differential equation $\frac{dy}{dx} + 2y = \sin x$.



Based on the above information, answer the following questions:

(i) What type of differential Equation is it? Find its order and degree.

(ii) Find the general solution of the differential equation of a curve along which the kite is flying.

38. Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$

- Right land Derivative (R.H.D.) : $Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = 0 if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$

Answer the following questions :

(i) What is R.H.D. of f(x) at x = 1?

(ii) What is L.H.D. of f(x) at x = 1?

(iii) (a) Check if the function f(x) is differentiable at x = 1

OR

(b) Find $f'(2)$ and $f'(-1)$